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FLOW OF A NON-NEWTONIAN LIQUID IN THE GAP BETWEEN A ROTATING CYLINDER AND A PERMEABLE SURFACE WITH ROTOR GRANULATION

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The isothermal process of rotor granulation of a material having the properties of an anomalously viscous liquid is analyzed hydrodynamically.

One of the highly promising methods for processing of highly viscous media is rotor granulation. Rotor machines which combine the functions of a pump and a forming device are characterized by minimum deformation of the material being processed and permit granulation of highly filled heterogeneous systems. Rotor-type granulators are widely used for processing of pastelike materials, suspensions, and polymers in the pharmaceutical, food, and metallurgical industries, for production of plastics and rubber parts, in mineral fractionation, and a number of other chemical technology processes [1].

The available theoretical studies of material flow in granulators [1-3] contain inaccuracies in formulation of the boundary problem. Thus, for example, their authors assume that flow terminates in a minimal gap and that excess pressure is equal to zero. This corresponds to the Ardichvili concept for a roller process in which the flow occurs at zero matrix permeability [4].

The present study will attempt a hydrodynamic analysis of flow of a non-Newtonian (power-law) liquid in a rotor granulator corresponding to the Gaskell concept for roller processes [4, 5].

Formulation of the Problem. A diagram of the flow is shown in Fig. 1. The mass to be processed is fed into the working cavity between the rotor and matrix, is held by those parts and forced through the perforated matrix. In the general case the peripheral velocity of the roller U may not be equal to the translational velocity of the matrix W. We assume that the flow is two-dimensional, laminar, and steady-state. The medium is incompressible. Compared to viscous forces, inertial and mass forces are negligibly small. Commencing from the continuity equation we have $v_x \sim U + W$, $v_y \sim (U + W)h/L$, $L \gg h$, where L and h are the characteristic lengths along the x- and y-axes. Evaluation of the terms of the equations of motion yields $\partial v_x/\partial x \sim (U + W)/L$, $\partial v_x/\partial y \sim (U + W)/h$, $\partial v_y/\partial x \sim (U + W)\cdot h/L^2$. We take $\partial P/\partial y = 0$, i.e., $P = P(x)$. There is no slippage on the working surfaces. The matrix permeability does not depend on its velocity of motion and is characterized by an empirical dependence [1, 2]

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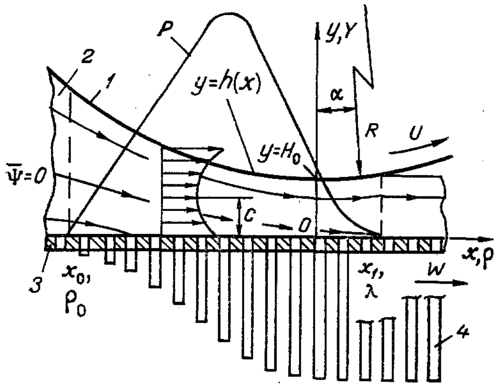

Fig. 1. Diagram of granulation process: 1) roller; 2) material being processed; 3) perforated matrix; 4) granulate.

$y = 0$, $v_y = -KP^{1/n}$, where K is an experimentally determined constant (which depends on the rheological properties of the medium and the geometry of the perforations).

Material flow in the working gap is described by the equations

$$\frac{dP}{dx} = \frac{\partial \tau_{xy}}{\partial y}, \tag{1}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \tag{2}$$

$$\tau_{xy} = \mu \left| \frac{\partial v_x}{\partial y} \right|^{n-1} \frac{\partial v_x}{\partial y}, \tag{3}$$

$$y = 0,\ v_x = W,\ v_y = -KP^{1/n}, \tag{4}$$

$$y = h,\ v_x = U,\ v_y = U\frac{x}{R}, \tag{5}$$

$$x = x_0,\ P = 0, \tag{6}$$

$$x = x_1,\ P = \frac{dP}{dx} = 0. \tag{7}$$

Condition (5) is obtained with the assumption that the velocity components on the roller surface $v_y = U \sin \alpha = Ux/R$, $v_x = U \cos \alpha \approx U$. The points x_0 and x_1 characterize the boundaries of the flow zone.

The problem of Eqs. (1)-(7) is underdefined, since for three of the unknown functions $P(x)$, $v_x(x, y)$, $v_y(x, y)$ we have only the two equations (1) and (2) [5, p. 234]. The coordinate x_1, being an a priori parameter of the problem uniquely defines the coordinate of the point x_1 and the unknown functions.

Integrating Eq. (2) over the region bounded by the sections x and x_1 and the lines $y = 0$ and $y = h(x)$, we obtain an integral continuity equation

$$\int_0^{h(x_1)} v_x(x_1)\, dy - \int_0^h v_x dy + K \int_x^{x_1} P^{1/n} dx = 0. \tag{8}$$

According to Eq. (8), the volume flow rate for a roller of unit width $\int_0^h v_x dy$ in an arbitrary section x is equal to the sum of the granulate flow rate $K\int_x^{x_1} P^{1/n} dx$ over the portion from x to x_1 and the flow rate at the output $\int_0^{h(x_1)} v_x(x_1)\, dy$.

Integrating Eq. (1) over y, we obtain an expression for the tangential stress

$$\tau_{xy}=\frac{dP}{dx}y+c_1(x). \tag{9}$$

It follows from simultaneous consideration of Eqs. (3) and (9) that

$$\frac{\partial v_x}{\partial y}=\left|\frac{y}{\mu}\frac{dP}{dx}+\frac{c_1}{\mu}\right|^{1/n}\operatorname{sign}\left(y\frac{dP}{dx}+c_1\right). \tag{10}$$

The further solution of the problem depends on the ratio of the working surface velocities U and W.

Equality of Working Surface Velocities. In the absence of friction (U = W) we have "symmetry" of the velocity profile

$$y=\frac{h}{2},\ \frac{\partial v_x}{\partial y}=0. \tag{11}$$

With consideration of Eq. (11), Eq. (10) takes on the form

$$\frac{\partial v_x}{\partial y}=\left|\frac{1}{\mu}\frac{dP}{dx}\left(y-\frac{h}{2}\right)\right|^{1/n}\operatorname{sign}\left[\frac{dP}{dx}\left(y-\frac{h}{2}\right)\right]. \tag{12}$$

For h we take a parabolic approximation [5]

$$h=H_0+\frac{x^2}{2R}. \tag{13}$$

We introduce the dimensionless variables and parameters

$$\rho=\frac{x}{\sqrt{2RH_0}},\ \rho_0=\frac{x_0}{\sqrt{2RH_0}},\ \lambda=\frac{x_1}{\sqrt{2RH_0}},$$
$$\bar{P}=\frac{PH_0^{n+1}}{\mu W^n\sqrt{2RH_0}},\quad \Gamma=\frac{K\sqrt{2RH_0}}{H_0^2}\left(\frac{\mu\sqrt{2RH_0}}{H_0}\right)^{1/n}, \tag{14}$$
$$Y=\frac{y}{h}=\frac{y}{H_0(1+\rho^2)},\ \bar{\Psi}=\frac{\Psi}{WH_0},\ f=\frac{U}{W}.$$

Integrating Eq. (12) with consideration of Eqs. (4), (5), (13), (14), we obtain an expression for the velocity components

$$\frac{v_x}{W}=1+A\left(|1-2Y|^{\frac{n+1}{n}}-1\right), \tag{15}$$

where

$$A(\rho)=\frac{n}{n+1}\left(\frac{1+\rho^2}{2}\right)^{\frac{n+1}{n}}\left|\frac{d\bar{P}}{d\rho}\right|^{1/n}\operatorname{sign}\left(\frac{d\bar{P}}{d\rho}\right).$$

For $A(\rho_0) > 1$ there is a stagnation point [5] in the flow zone, at which $v_x = 0$.

Substituting Eq. (15) in Eq. (8) with consideration of Eq. (14) and $v_x(x_1) = W$. We obtain a nonlinear first order integrodifferential equation for the dimensionless pressure

$$\lambda^2-\rho^2+\frac{n}{2(2n+1)}\left|\frac{(1+\rho^2)^{2n+1}}{2}\frac{d\bar{P}}{d\rho}\right|^{1/n}\operatorname{sign}\left(\frac{d\bar{P}}{\partial\rho}\right)+\Gamma\int_\rho^\lambda\bar{P}^{1/n}d\rho=0. \tag{16}$$

Boundary conditions (6), (7) for the dimensionless pressure and unknown parameters ρ_0 and λ have the form

$$\rho=\rho_0,\ \bar{P}=0,\ \rho=\lambda,\ \bar{P}=\frac{d\bar{P}}{d\rho}=0. \tag{17}$$

Numerical analysis of Eqs. (16), (17) was performed by Euler's method, beginning at the point $\rho = \lambda$, which avoids use of the firing method [1, 3] used traditionally for this problem. In finite differences Eq. (16) has the form

$$\bar{P}_{m+1} = \bar{P}_m - \frac{2\Delta\rho}{(1+\rho^2)^{2n+1}} \left[\frac{2(2n+1)|\rho_m^2 - \lambda^2 - \Gamma I_m|}{n}\right]^n \operatorname{sign}(\rho_m^2 - \lambda^2 - \Gamma I_m), \tag{18}$$

where $\rho_m = \lambda - \Delta\rho m$; the integral $I_m = \int\limits_{\rho_m}^{\lambda} \bar{P}^{1/n} d\rho$ is defined by the trapezoid formula:

$$I_{m+1} = I_m + \frac{\Delta\rho}{2} (\bar{P}_m^{1/n} + \bar{P}_{m+1}^{1/n}). \tag{19}$$

The initial conditions for system (18), (19) has the form: $m = 0$, $\rho_m = \lambda$, $I_m = 0$, $\bar{P}_m = 0$. Calculations are halted at $\bar{P}_{m+1} < 0$. Results of the analysis of Eqs. (18), (19) are shown in Fig. 2. Calculations were performed with a step $\Delta\rho = 5 \cdot 10^{-3}$ for $\lambda = 0.3$. It is evident from the figure that to produce an identical thickness of processed material on the matrix (λ = const) the extent of the flow zone ($\rho_0 - \lambda$) for a pseudoplastic liquid must be larger than for a Newtonian or dilatant liquid. With increase in matrix permeability the size of the flow zone grows and pressure increases. The calculated curves correspond qualitatively to the experimental results of Sigaev [1].

Using the permeability condition we can define the granulate output

$$Q = b \int\limits_{x_0}^{x_1} |v_y(y=0)|\, dx = bK \int\limits_{x_0}^{x_1} P^{1/n} dx,$$

or with consideration of Eq. (14)

$$\bar{Q} = \frac{Q}{bH_0W} = \Gamma \int\limits_{\rho_0}^{\lambda} \bar{P}^{1/n} d\rho. \tag{20}$$

The tensile force is defined by the integral

$$F = b \int\limits_{x_0}^{x_1} P dx,$$

or with consideration of Eq. (14),

$$\bar{F} = \frac{F}{2bR\mu} \left(\frac{H_0}{W}\right)^n = \int\limits_{\rho_0}^{\lambda} \bar{P} d\rho. \tag{21}$$

The power expended in displacement of the matrix and rotation of the roller is found from the integral

$$N = b \int\limits_{x_0}^{x_1} (\tau_{xy} v_x)|_0^h dx.$$

Considering Eqs. (3), (12), (14), we can write

$$\bar{N} = \frac{NH_0^n}{b\mu \sqrt{2RH_0}\, W^{n+1}} = -2 \int\limits_{\rho_0}^{\lambda} \bar{P}\rho d\rho. \tag{22}$$

Using the equation for the flow function $v_x = \partial\Psi/\partial y$ and Eq. (15), and taking the boundary condition on the roller surface $Y = 1$, $\bar{\Psi} = 1 + \lambda^2$, we can find an expression for the flow function:

$$\bar{\Psi} = 1 + \lambda^2 - (1+\rho^2) \left\{\frac{An}{2(2n+1)} [1 + |1-2Y|^{\frac{2n+1}{n}} \operatorname{sign}(1-2Y)] + (1-A)(1-Y)\right\}. \tag{23}$$

The line $\bar{\Psi} = 0$ passes through the point $\rho = \lambda$, $Y = 0$ and divides the entire flow region into a region of particle trajectories passing through the matrix ($\bar{\Psi} < 0$), and a region of liquid particle trajectories which remain on the matrix surface ($\bar{\Psi} > 0$), as shown in Fig. 1.

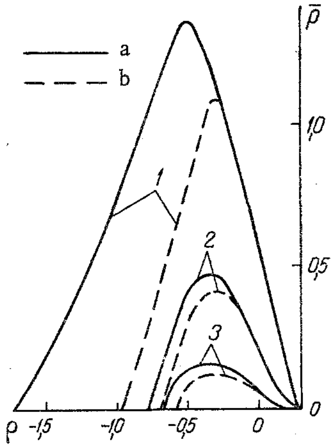

Fig. 2. Dimensionless pressures: a) Γ = 0.2; b) Γ = 0; 1) n = 0.2; 2) 1; 3) 2.

Differing Working Surface Velocities. In the presence of friction (f ≠ 1) the expression for the velocity v_x obtained by integrating Eq. (10) with consideration of condition (4) has the form

$$v_x = W + \left|\frac{h^{1+n}}{\mu}\frac{dP}{dx}\right|^{1/n} \operatorname{sign}\left(\frac{dP}{dx}\right)\int_0^Y |Y-C|^{1/n}\operatorname{sign}(Y-C)\,dY, \tag{24}$$

where $C = -C_1\Big/\left(h\frac{dP}{dx}\right)$. The function C(x) characterizes the dimensionless ordinate of the point of maximum velocity $\partial v_x(Y = C)/\partial Y = 0$ (see Fig. 1).

Performing the integration in Eq. (24), we find

$$v_x = W + \frac{n}{n+1}\left|\frac{h^{1+n}}{\mu}\frac{dP}{dx}\right|^{1/n} \operatorname{sign}\left(\frac{dP}{dx}\right)\left(|Y-C|^{\frac{n+1}{n}} - |C|^{\frac{n+1}{n}}\right). \tag{25}$$

Using condition (5) for Eq. (25) and transforming to the dimensionless variables of Eq. (14), we obtain

$$f-1 = \frac{n}{n+1}B(1+\rho^2)^{\frac{n+1}{n}}\left|\frac{d\bar{P}}{d\rho}\right|^{1/n}\operatorname{sign}\left(\frac{d\bar{P}}{d\rho}\right), \tag{26}$$

where $B = |1-C|^{\frac{n+1}{n}} - |C|^{\frac{n+1}{n}}$.

Equation (26) allows elimination of the pressure gradient from Eq. (25)

$$\frac{v_x}{W} = 1 + B^{-1}(f-1)\left(|Y-C|^{\frac{n+1}{n}} - |C|^{\frac{n+1}{n}}\right). \tag{27}$$

The axial velocity profile at the output is given by solution of Eq. (10) with condition (7):

$$v_x(x_1) = W + \frac{U-W}{h(x_1)}y, \tag{28}$$

where $h(x_1) = H_0(1 + \lambda^2)$.

Substituting Eqs. (27), (28) in Eq. (8) and considering Eqs. (13), (14), we obtain

$$0{,}5(1+f)(1+\lambda^2) - (1+\rho^2)[1+(f-1)\Phi] + \Gamma\int_\rho^\lambda \bar{P}^{1/n}\,d\rho = 0, \tag{29}$$

where $\Phi = \frac{n}{2n+1}B^{-1}\left\{|C|^{\frac{n+1}{n}}\left[C-\left(\frac{2n+1}{n}\right)\right] - |C-1|^{\frac{2n+1}{n}}\operatorname{sign}(C-1)\right\}$. The function Φ(C) has two

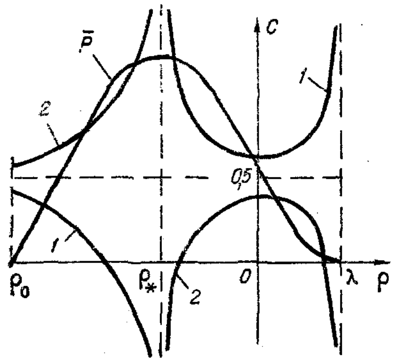

Fig. 3. Function C vs friction: 1) f > 1; 2) f < 1.

asymptotes, C = 0.5 and Φ = 0.5 with no extrema or inflection points. For $-\infty < C < 0.5$ the function decreases from 0.5 to $-\infty$, while for $0.5 < C < +\infty$ it decreases from $+\infty$ to 0.5.

Thus, for the two unknown functions C and $\overline{P}$ we have the system of Eqs. (26), (29) and boundary conditions (17).

In the particular case n = 1, $\Phi = (1 - 3C)/3(1 - 2C)$ and we can eliminate C from the system (26), (29), obtaining for the dimeneionless pressure the equation

$$0{,}5\,(f+1)(\lambda^2-\rho^2)+\frac{1}{12}(1+\rho^2)^3\frac{d\overline{P}}{d\rho}+\Gamma\int\limits_{\rho}^{\lambda}\overline{P}d\rho=0. \quad (30)$$

Analysis of Eq. (30) reveals that with increase in friction the size of the flow zone and maximum pressure value increase. Moreover the granulate output increases and the point of maximum pressure (in the notation of Fig. 1) shifts leftward.

The character of the change in C as a function of friction is illustrated by Fig. 3. It is evident from the figure that at f < 1 (f > 1) the function C increases (decreases) from C > 0.5 (C < 0.5) to $+\infty$ $(-\infty)$, suffers an infinite discontinuity at the point ρ_* changing from $-\infty$ to $-\infty$ (from $+\infty$ to $+\infty$) through a maximum (minimum) at which C < 0.5 (C > 0.5).

At the stagnation point the condition $v_x = 0$, Y = C is satisfied and from Eq. (27) for C we have $C_*=1/(1+f^{\frac{n}{n+1}})$. Consequently, liquid circulation at the input occurs at $C(\rho_0) < C_*$ for f < 1 or $C(\rho_0) > C_*$ for f > 1.

The system of equations (26), (29) can be conveniently solved numerically analogously to Eq. (16), beginning at the point $\rho = \lambda$. The output and tensile force are determined by Eqs. (20), (21). The required power is determined in analogy to Eq. (21) and can be found as

$$\overline{N}=\left[\frac{(n+1)\,|f-1|}{n}\right]^n\int\limits_{\rho_0}^{\lambda}|B\,(1+\rho^2)|^{-n}\,[f-C\,(f-1)]\,\mathrm{sign}\,[B(f-1)]\,d\rho.$$

The mathematical models obtained uniquely relate the integral parameters of the process (required power, tensile force, granulate output) to the dimensionless parameter λ, which characterizes the thickness $H_0(1 + \lambda^2)$ or flow rate of the material at the output $0.5\,H_0 b(U + W)(1 + \lambda^2)$. Therefore if we represent the dependence $\overline{N} = \overline{N}(\Gamma, \lambda, f, n)$, $\overline{F} = \overline{F}(\Gamma, \lambda, f, n)$, $\overline{Q} = \overline{Q}(\Gamma, \lambda, f, n)$ in the form of nomograms, by specifying the value of λ and knowing the other parameters appearing in Γ and f we can find N, F, and Q.

The analysis presented here can be extended to the case of liquid flow in a curvilinear gap between two rotating cylinders with radii R_1 and R_2, one of which is solid, the other perforated. The effective radius of curvature appearing in Eq. (14) is then given by the expression $R = R_1R_2/|R_1 \pm R_2|$. The minus sign is for the case where the cylinder axes are both to one side of the flow zone (x-axis) while the plus sign is used when the cylinders are on opposite sides of the flow zone. The cylinder axes are parallel.

NOTATION

x, y, Cartesian coordinates; U, roller peripheral velocity; W, matrix translational velocity; h, current gap height; L, characteristic length of flow zone; v_x, v_y, velocity components; K, matrix permeability coefficient; P, $\bar{P}$, dimensional and dimensionless pressures; μ, n, rheological constants; τ_{xy}, shear stress; R, roller radius; x_0, x_1, mixing zone characteristic points; ρ, dimensionless Gaskell variable; ρ_0, λ, coordinates of flow zone boundary; Γ, dimensionless matrix permeability; Y, dimensionless ordinate; Ψ, $\bar{\Psi}$, dimensional and dimensionless flow functions; f, friction; A, a function of ρ; m, number of point; I(ρ), integral function; Δρ, step in ρ; Q, volume granulate output; F, tensile force; N, required power; C, a function of ρ; C_*, value of function C at stagnation point; ρ_*, dimensionless coordinate of pressure maximum; c_1, a function of x; α, central angle; $\bar{Q}$, $\bar{F}$, $\bar{N}$, dimensionless output, tensile force, and required power; b, working width of roller.

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APPROXIMATION OF THE GENERALIZED BUCKINGHAM EQUATION

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The generalized Buckingham equation is approximated by a quadratic function for media describable by the Balkley–Gershel model.

The Balkley–Gershel equation of state [1] is a quite general rheological law which describes the behavior of various high concentration suspensions:

$$\tau = \tau_0 \operatorname{sign} \frac{du}{dr} + k \left| \frac{du}{dr} \right|^{n-1} \frac{du}{dr}.$$

For a laminar flow regime in a circular tube the relationship between volume flow rate and friction on the wall is defined by the generalized Buckingham equation [2]

$$Q = \pi R^3 \frac{n}{3n+1} \left(\frac{\tau_w}{k} \right)^{1/n} (1 - \bar{r}_p)^{\frac{n+1}{n}} \left[1 + \frac{2n}{2n+1} \bar{r}_p + \frac{2n^2}{(n+1)(2n+1)} \bar{r}_p^2 \right]. \tag{1}$$

Use of Eq. (1) in practical calculations is difficult, since it is usually necessary to define the pressure drop ΔP in terms of the volume flow rate, i.e., ΔP = f(Q).

We will approximate the auxiliary function $\varphi_1(x)$ by the quadratic expression $\varphi_2(x)$:

$$\varphi_1(x) = (1 - x)^{n+1} \left(1 + \frac{2n}{2n+1} x + \frac{2n^2}{(n+1)(2n+1)} x^2 \right)^n, \tag{2}$$

$$\varphi_2(x) = (ax^2 + bx + c)^{1/2} + d + ex. \tag{3}$$

It is evident that the expression $\varphi_1^{1/n}(x)$ coincides with the terms in square brackets in Eq. (1).

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